ECON 7670: Efficiency

Elliott Isaac

Department of Economics
Tulane University

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Efficiency in Theory

Efficiency

- Incidence: effect of policies on **distribution** of economic pie
  - Focus is on prices
- Efficiency/deadweight loss/excess burden: effect of policies on **size** of the pie
  - Focus is on quantities

Efficiency

- Government raises taxes for one of two reasons:
  1. To raise revenue to finance public goods
  2. To redistribute income
- But to generate $1 of revenue, welfare of those taxed falls by more than $1 because the tax distorts behavior
- How to implement policies that minimize these efficiency costs?
  - Start with positive analysis of how to measure efficiency cost of a given tax system

Efficiency

- Simplest analysis of efficiency costs: Marshallian surplus
- Two assumptions:
  1. Quasilinear utility: no income effects, money metric
  2. Perfectly competitive production

Model Setup

- Two goods: x and y
- Consumer has wealth Z, utility $u(x)+y$, and solves

$$\max_{x,y} u(x)+y$$
$$\text{s.t. } (p+\tau)x(p+\tau,Z)+y(p+\tau,Z)=Z$$

- Firms use $c(S)$ units of the numeraire y to produce S units of x
- Marginal cost of production is increasing and convex:

$$c'(S)>0 \text{ and } c''(S)\geq 0$$

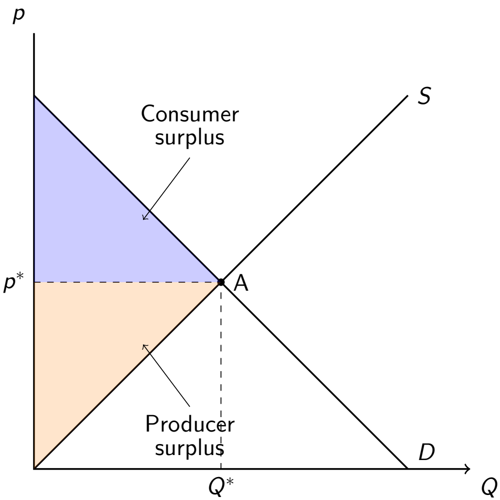
- Firm's profit at pretax price p and level of supply S is

$$pS-c(S)$$

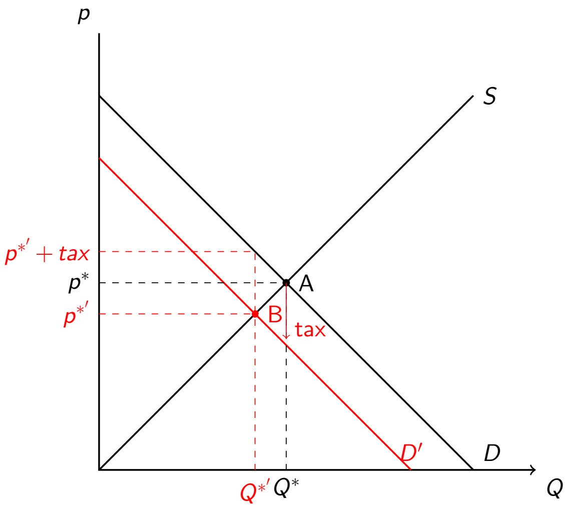
Model Equilibrium

- With perfect optimization, supply function for x ($S(p)$) is implicitly defined by the FOC
$$p = c'(S(p))$$
- Let $\eta_S = \frac{\partial S(p)}{\partial p} \frac{p}{S(p)} = p\frac{S'}{S}$ denote the price elasticity of supply
- Let Q denote equilibrium quantity sold of good x
- Q satisfies:
$$Q(t) = D(p+t) = S(p)$$
- Consider effect of introducing a small tax $dt > 0$ on Q and surplus

Deadweight Loss



Deadweight Loss



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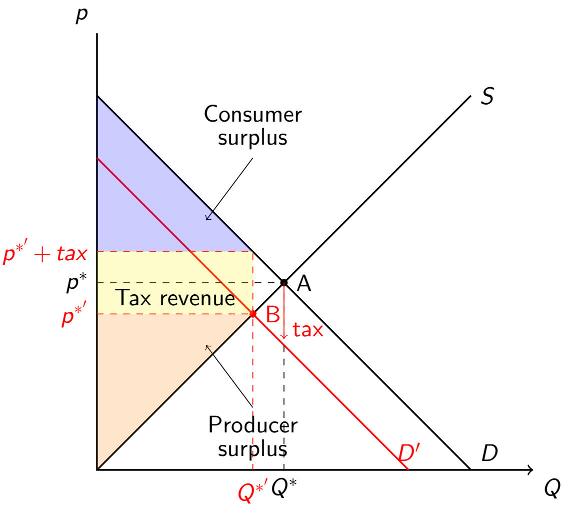

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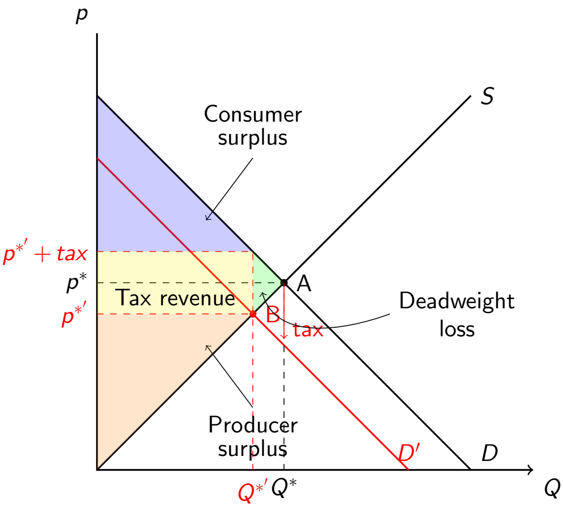

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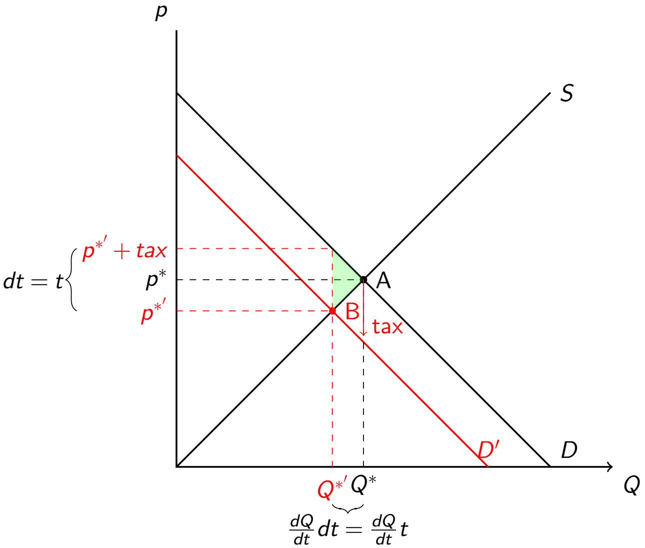


Deadweight Loss

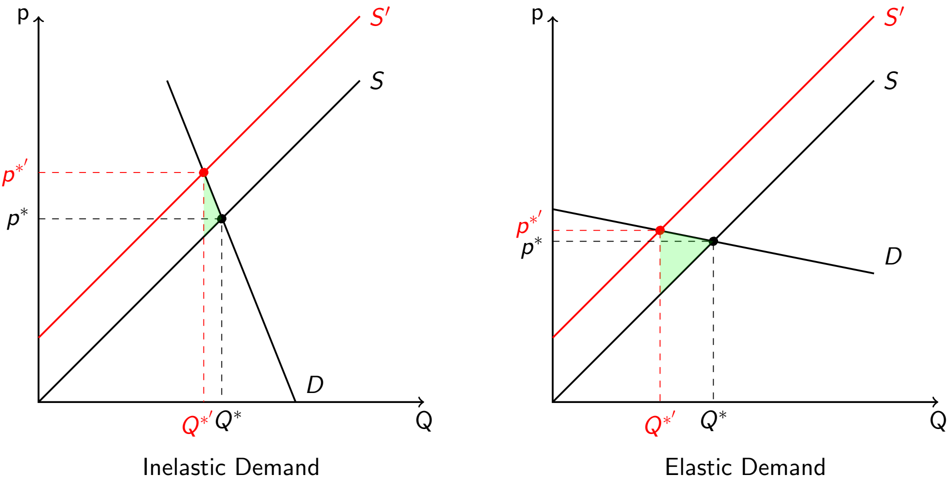
Qualitative properties of deadweight loss:

1. Deadweight loss increases with square of tax rate
   - Height of DWL triangle is t
   - Width of DWL triangle is $\frac{dQ}{dt}t$
   - $\Rightarrow \text{DWL} = \frac{1}{2}\left(\frac{dQ}{dt}t^2\right)$

Deadweight Loss

Qualitative properties of deadweight loss

2. Deadweight loss increases with elasticities



Tax Policy Implications

- With many goods, the most efficient way to raise tax revenue is:
  1. Tax inelastic goods more (e.g. medical drugs, food)
  2. Spread taxes across all goods to keep tax rates relatively low on all goods (broad tax base)
- These are two countervailing forces; balancing them requires quantitative measurement of deadweight loss

Measuring Deadweight Loss

- How can we measure and estimate deadweight loss empirically?
- Three empirically implementable methods depending on what data you have access to:
  1. In terms of supply and demand elasticities
  2. In terms of total change in equilibrium quantity caused by tax
  3. In terms of change in government revenue

Measuring Deadweight Loss: Supply & Demand Elasticities

$$\begin{aligned}
DWL &= -\frac{1}{2}\frac{dQ}{dt}(dt)^2 \\
DWL &= -\frac{1}{2}dQdt \\
DWL &= -\frac{1}{2}\frac{\partial S(p)}{\partial p}dpdt \\
DWL &= \frac{1}{2}\frac{\partial S(p)}{\partial p}\left(\frac{\varepsilon_D}{\varepsilon_S-\varepsilon_D}dt\right)dt \\
DWL &= \frac{1}{2}\left(\frac{\partial S(p)}{\partial p}\frac{p}{S(p)}\right)\frac{S(p)}{p}\frac{p}{p}\frac{\varepsilon_D}{\varepsilon_S-\varepsilon_D}dt^2 \\
DWL &= \frac{1}{2}\frac{\varepsilon_S\varepsilon_D}{\varepsilon_S-\varepsilon_D}pQ\left(\frac{dt}{p}\right)^2
\end{aligned}$$

- Note: second line uses $\frac{dQ(t)}{dt}dt = \frac{\partial S(p)}{\partial p}\frac{dp}{dt}dt = \frac{\partial S(p)}{\partial p}dp$
- Note: third line uses incidence formula $dp = \frac{\varepsilon_D}{\varepsilon_S-\varepsilon_D}dt$

Measuring Deadweight Loss: Supply & Demand Elasticities

$$DWL = \frac{1}{2}\frac{\varepsilon_S \varepsilon_D}{\varepsilon_S - \varepsilon_D} pQ \left(\frac{dt}{p}\right)^2$$

- What data do you need to estimate DWL using this method?

Measuring Deadweight Loss: Supply & Demand Elasticities

$$DWL = \frac{1}{2}\frac{\varepsilon_S \varepsilon_D}{\varepsilon_S - \varepsilon_D} pQ \left(\frac{dt}{p}\right)^2$$

- What data do you need to estimate DWL using this method?
  - Price
  - Quantity
  - Tax change
  - A way to separately identify and estimate ε_s and ε_D

Measuring Deadweight Loss: Supply & Demand Elasticities

- Can simplify the previous equation by recognizing that tax revenue $R = Qdt$

- Useful expression is deadweight loss per dollar of tax revenue:

$$\frac{DWL}{R} = \frac{1}{2}\frac{\varepsilon_S \varepsilon_D}{\varepsilon_S - \varepsilon_D}\frac{dt}{p}$$

- Now you do not need quantity to estimate $\frac{DWL}{R}$

Measuring Deadweight Loss: Distortions in Equilibrium Q^*

- Define $\eta_Q = -\frac{dQ}{dt}\frac{p_0}{Q}$
- η_Q: effect of a 1% increase in price via a tax change on equilibrium quantity, taking into account the endogenous price change
- This is the coefficient β in a reduced-form regression:

$$\log Q = \alpha + \beta \frac{t}{p_0} + \varepsilon$$

- Identify β using exogenous variation in t. Then:

$$\begin{aligned} DWL &= -\frac{1}{2}\frac{dQ}{dt}(dt)^2 \\ DWL &= -\frac{1}{2}\frac{dQ}{dt}\left(\frac{p}{Q}\right)\left(\frac{Q}{p}\right)(dt)^2 \\ DWL &= \frac{1}{2}\eta_Q pQ\left(\frac{dt}{p}\right)^2 \end{aligned}$$

Measuring Deadweight Loss: Distortions in Equilibrium Q^*

$$DWL = \frac{1}{2}\eta_Q pQ\left(\frac{dt}{p}\right)^2$$

- What data do you need to estimate DWL using this method?
  - Price
  - Quantity
  - Tax change
  - A way to identify and estimate $\beta = \eta_Q$

Measuring *Marginal* Deadweight Loss Due to Tax Increases

- Deadweight loss of instituting a tax t is

$$DWL(t) = -\frac{1}{2}\frac{dQ}{dt}t^2$$

- Consider DWL from raising tax by Δt given pre-existing tax t:

$$\begin{aligned} DWL(\Delta t) &= -\frac{1}{2}\frac{dQ}{dt}[(t+\Delta t)^2 - t^2] \\ DWL(\Delta t) &= -\frac{1}{2}\frac{dQ}{dt}\cdot[2t\cdot\Delta t + (\Delta t)^2] \\ DWL(\Delta t) &= -t\frac{dQ}{dt}\Delta t - \frac{1}{2}\frac{dQ}{dt}(\Delta t)^2 \end{aligned}$$

- First term is first-order in Δt; second term is second-order ($(\Delta t)^2$)
- This is why taxing markets with pre-existing taxes generates larger marginal DWL
  - DWL of $\Delta t = 1\%$ is 10 times larger if $t = 10\%$ than if $t = 0$.

Measuring *Marginal* Deadweight Loss Due to Tax Increases

- Computing marginal DWL by differentiating formula for DWL gives:

$$\frac{dDWL}{dt}\Delta t = -t\frac{dQ}{dt}\Delta t$$

- First derivative of $DWL(t)$ only includes first-order term in Taylor expansion:

$$\begin{aligned} DWL(t+\Delta t) &= DWL(t) + \frac{dDWL}{dt}\Delta t + \frac{1}{2}\frac{d^2DWL}{dt^2}(\Delta t)^2 \\ \Rightarrow DWL(t+\Delta t) - DWL(t) &= \frac{dDWL}{dt}\Delta t + \frac{1}{2}\frac{d^2DWL}{dt^2}(\Delta t)^2 \end{aligned}$$

- First-order approximation is accurate when t large relative to Δt
  - Ex: $t = 20\%$, $\Delta t = 5\%$ implies first term accounts for 90% of DWL
  - But introduction of new tax ($t = 0$) generates DWL only through second-order term

Measuring *Marginal* Deadweight Loss Due to Tax Increases

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Measuring Deadweight Loss: Leakage in Government Revenue

- To first order, marginal excess burden of raising τ is:

$$\frac{\partial DWL}{\partial t} = -t\frac{dQ}{dt}$$

- Observe that tax revenue $R(t) = Qt$
  - Mechanical revenue gain: $\left.\frac{\partial R}{\partial t}\right|_Q = Q$
  - Actual revenue gain: $\frac{\partial R}{\partial t} = Q + t\frac{dQ}{dt}$
- MDWL is the difference between mechanical and actual revenue gain:

$$\left.\frac{\partial R}{\partial t}\right|_Q - \frac{dR}{dt} = Q - \left[Q + t\frac{dQ}{dt}\right] = -t\frac{dQ}{dt} = \frac{\partial DWL}{\partial t}$$

Measuring Deadweight Loss: Leakage in Government Revenue

$$\left.\frac{\partial R}{\partial t}\right|_{Q} - \frac{dR}{dt} = Q - \left[Q + t\frac{dQ}{dt}\right] = -t\frac{dQ}{dt} = \frac{\partial DWL}{\partial t}$$

- What data do you need to estimate marginal DWL using this method?
  - Anticipated tax revenue gain (or Q)
  - Actual tax revenue gain

Measuring Deadweight Loss: Leakage in Government Revenue

- Why does leakage in govt. revenue only capture first-order term?
  - Govt revenue loss: rectangle in Harberger trapezoid, proportional to Δt
  - Consumer and producer surplus loss: triangles in trapezoid (proportional to $(\Delta t)^2$)
- Leakage approach is accurate for measuring marginal excess burden given pre-existing taxes but not introduction of new taxes

Skip General Model

General Model with Income Effects

General Model with Income Effects

- Drop quasilinearity assumption and consider an individual with utility

$$u(c_1, .., c_N) = u(c)$$

- Individual's problem:

$$\max_c u(c)$$
$$\text{s.t. } (p+t)c \leq Z$$

where $p+t$ denotes vector of tax-inclusive prices and Z is wealth

- Labor can be viewed as commodity with price w and consumed in negative quantity

General Model with Income Effects

- Let λ denote multiplier on budget constraint
- First order condition in c_i:

$$u_{c_i} = \lambda q_i$$

- These conditions implicitly define:
  - $c_i(p+t, Z)$: the Marshallian ("uncompensated") demand function
  - $v(p+t, Z)$: the indirect utility function

Measuring Deadweight Loss with Income Effects

- Question: how much utility is lost because of tax beyond revenue transferred to government?
- Marshallian surplus does not answer this question with income effects
  - Problem: not derived from utility function or a welfare measure
  - Creates various problems such as "path dependence" with taxes on multiple goods

$$\Delta CS(t^0 \to \tilde{t}) + \Delta CS(\tilde{t} \to t^1) \neq \Delta CS(t^0 \to t^1)$$

- Need units to measure "utility loss"
  - Introduce expenditure function to translate the utility loss into dollars (money metric)

Expenditure Function

- Fix utility at U and prices at p
- Find bundle that minimizes cost to reach U for p:

$$e(p, U) = \min_c p \cdot c$$
$$\text{s.t. } u(c) \geq U$$

- Let μ denote multiplier on utility constraint
- First order conditions given by:

$$p_i = \mu u_{c_i}$$

- These implicitly define Hicksian ("compensated") demand functions:

$$c_i = h_i(p, u)$$

- Define individual's loss from tax increase as

$$e(p^1, u) - e(p^0, u)$$

- Single-valued function $\rightarrow$ coherent measure of welfare cost, no path dependence

Compensating and Equivalent Variation

- But where should u be measured?
- Consider a price change from p^0 to p^1
- Utility at initial price p^0:

$$u^0 = v(p^0, Z)$$

- Utility at new price p^1:

$$u^1 = v(p^1, Z)$$

- Two concepts: compensating variation (CV) uses u^0 and equivalent variation (EV) uses u^1 as reference utility levels

Compensating Variation

- Measures utility at initial price level (u^0)
- Amount agent must be compensated in order to be indifferent about tax increase

$$CV = e(p^1, u^0) - e(p^0, u^0) = e(p^1, u^0) - Z$$

- How much compensation is needed to reach original utility level at *new* prices?
- CV is amount of ex-post cost that must be covered by government to yield same *ex-ante* utility:

$$e(p^0, u^0) = e(p^1, u^0) - CV$$

Equivalent Variation

- Measures utility at new price level
- Lump sum amount agent willing to pay to avoid tax (at pre-tax prices)

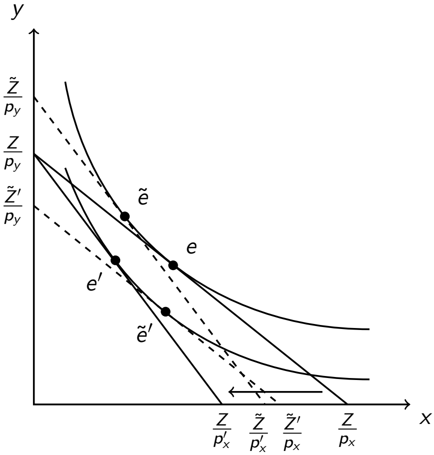
$$EV = e(p^1, u^1) - e(p^0, u^1) = Z - e(p^0, u^1)$$

- EV is amount extra that can be taken from agent to leave him with same *ex-post* utility:

$$e(p^0, u^1) + EV = e(p^1, u^1)$$

Compensating and Equivalent Variation with 2 Goods

- Good x is taxed, Good y is not taxed



- Compensating variation is $\tilde{Z} - Z$
- Equivalent variation is $Z - \tilde{Z}'$

Deadweight Loss with Income Effects

- Goal: derive empirically implementable formula analogous to Marshallian DWL formula in general model with income effects
- Literature typically assumes either
  1. Fixed producer prices and income effects
  2. Endogenous producer prices and quasilinear utility
- With both endogenous prices and income effects, efficiency cost depends on how profits are returned to consumers
- Formulas are very messy and fragile (Auerbach 1985, Section 3.2)

Deadweight Loss with Income Effects

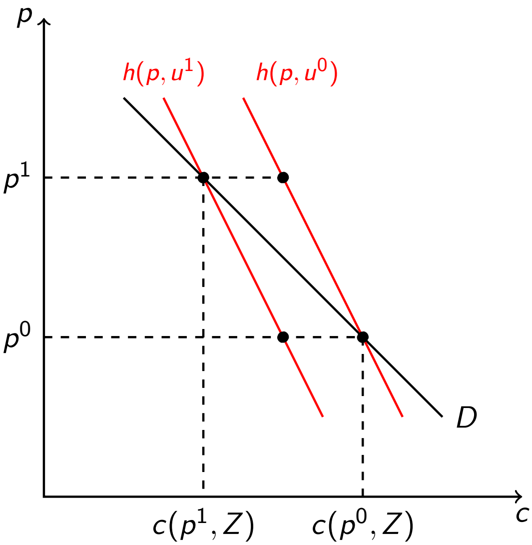
- Goal: derive empirically implementable formulas using Hicksian demand (EV and CV)
- Assume p is fixed $\rightarrow$ flat supply, constant returns to scale
- The envelope theorem implies that $e_{p_i}(p, u) = h_i$, and so:

$$e(p^1, u) - e(p^0, u) = \int_{p^0}^{p^1} h(p, u) dp$$

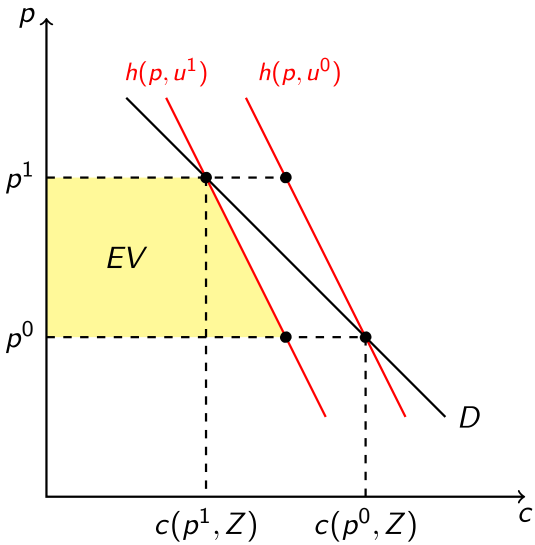
- If only one price is changing, this is the area under the Hicksian demand curve for that good
- Note that optimization implies that

$$h(p, v(p, Z)) = c(p, Z)$$

Deadweight Loss with Income Effects



Deadweight Loss with Income Effects



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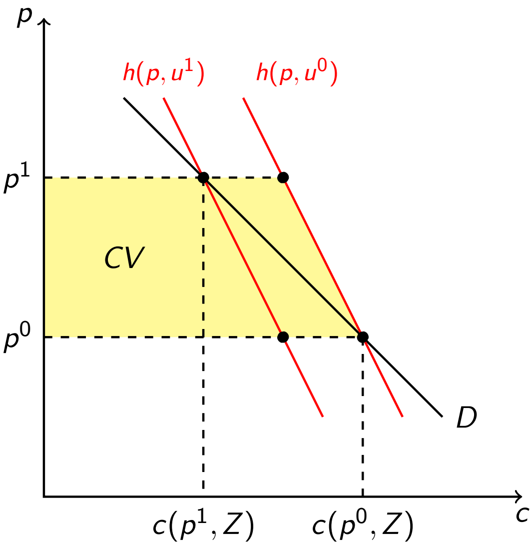

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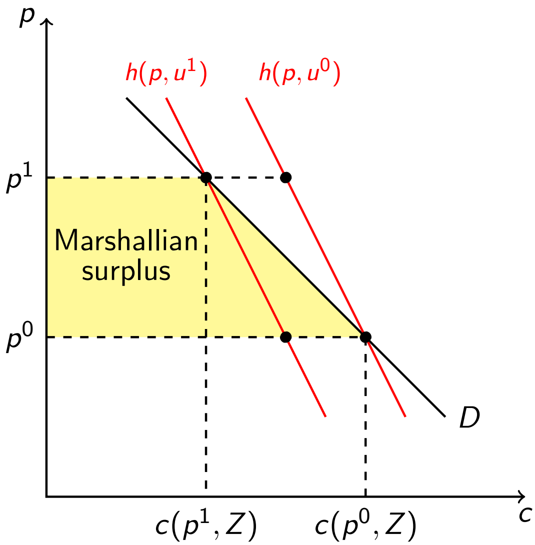


EV, CV, and Marshallian Surplus

- With one price change:

$$EV < \text{Marshallian Surplus} < CV$$

- But this is not true in general with multiple price changes because Marshallian Surplus is ill-defined

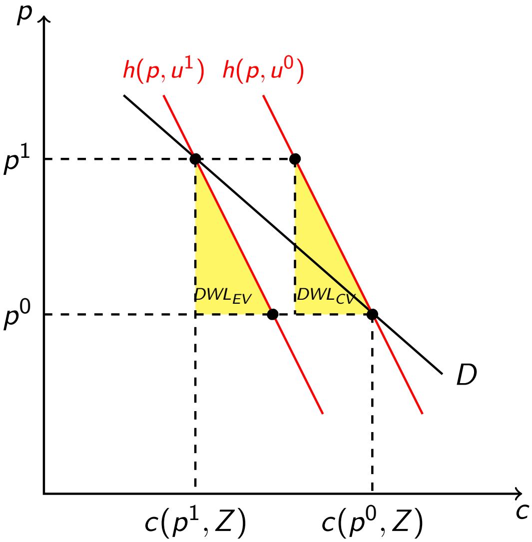
Deadweight Loss with Income Effects

- Deadweight loss: change in consumer surplus less tax paid
- What is lost in excess of taxes paid?
- Two measures, corresponding to EV and CV:

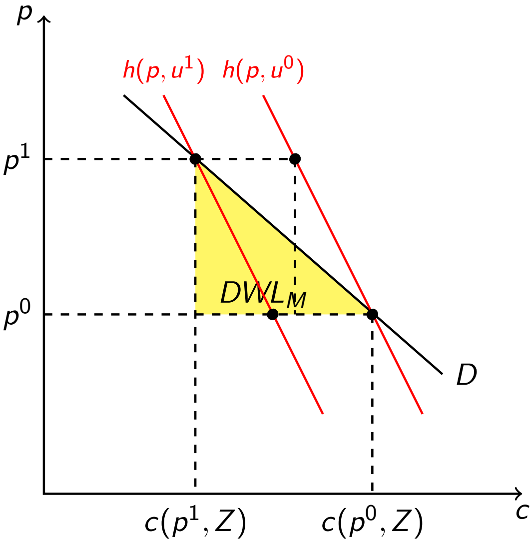
$$\begin{aligned} DWL(u^1) &= EV - (p^1 - p^0)h(p^1, u^1) \\ &\quad \text{[Mohring 1971]} \\ DWL(u^0) &= CV - (p^1 - p^0)h(p^1, u^0) \\ &\quad \text{[Diamond and McFadden 1974]} \end{aligned}$$

- Note that $p^1 - p^0 = t$

Deadweight Loss with Income Effects



Deadweight Loss with Income Effects



Deadweight Loss with Income Effects

- In general, CV and EV measures of DWL will differ
- Marshallian measure overstates DWL because it includes income effects
  - Income effects are not a distortion in transactions
  - Buying less of a good due to having less income is not an efficiency loss; no surplus foregone b/c of transactions that do not occur
- CV=EV=Marshallian DWL only with quasilinear utility (Chipman and Moore 1980)

Harberger Formula

- Consider increase in tax t on good 1 to $t + \Delta t$
- No other taxes in the system
- Recall the expression for initial DWL:

$$DWL(t) = [e(p+t,U) - e(p,U)] - th(p+t,U)$$

- Second-order Taylor expansion:

$$\begin{aligned} MDWL &= DWL(t+\Delta t) - DWL(t) \\ &\simeq \frac{dDWL}{dt}\Delta t + \frac{1}{2}(\Delta t)^2 \frac{d^2 DWL}{dt^2} \end{aligned}$$

Harberger Formula

- What are $\frac{dDWL}{dt}$ and $\frac{d^2DWL}{dt^2}$?

$$\begin{aligned}
\frac{dDWL}{dt} &= h_1(p+t,U) - t\frac{dh_1}{dt} - h_1(p+t,U) \\
&= -t\frac{dh_1}{dt} \\
\frac{d^2DWL}{dt^2} &= -\frac{dh_1}{dt} - t\frac{d^2h_1}{dt^2}
\end{aligned}$$

- Standard practice in literature: assume $\frac{d^2h_1}{dt^2} = 0$ (linear Hicksian); not necessarily well justified b/c it does not vanish as $\Delta t \to 0$

$$\Rightarrow MDWL = -t\Delta t\frac{dh_1}{dt} - \frac{1}{2}\frac{dh_1}{dt}(\Delta t)^2$$

- Formula equals area of "Harberger trapezoid" using Hicksian demands

Harberger Formula

- Without pre-existing tax, obtain "standard" Harberger formula:

$$DWL = -\frac{1}{2}\frac{dh_1}{dt}(\Delta t)^2$$

- General lesson: use compensated (substitution) elasticities to compute DWL, not uncompensated elasticities
- To estimate compensated elasticities empirically, estimate Marshallian price elasticity and income elasticity. Then apply Slutsky equation:

$$\underbrace{\frac{\partial h_i}{\partial p_j}}_{\substack{\text{Hicksian}\\ \text{Slope}}} = \underbrace{\frac{\partial c_i}{\partial p_j}}_{\substack{\text{Marshallian}\\ \text{Slope}}} + \underbrace{c_j \frac{\partial c_i}{\partial Z}}_{\substack{\text{Income}\\ \text{Effect}}}$$

Ito (2014)

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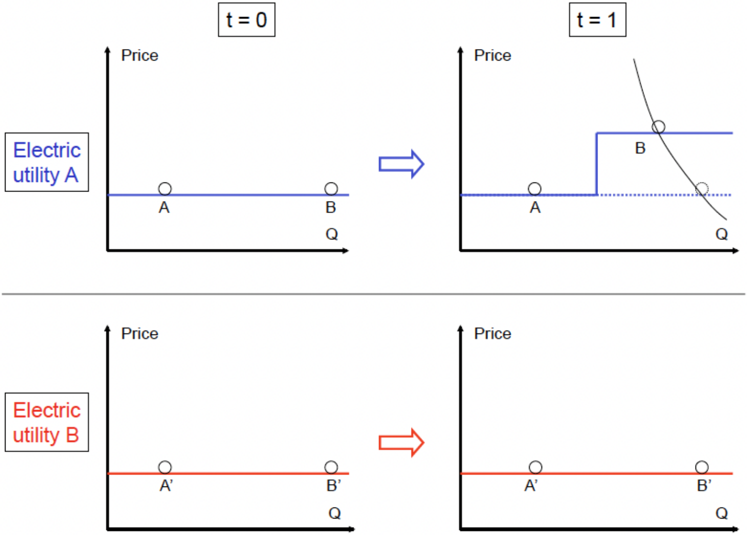
- Illustration of identification



Ito (2014)

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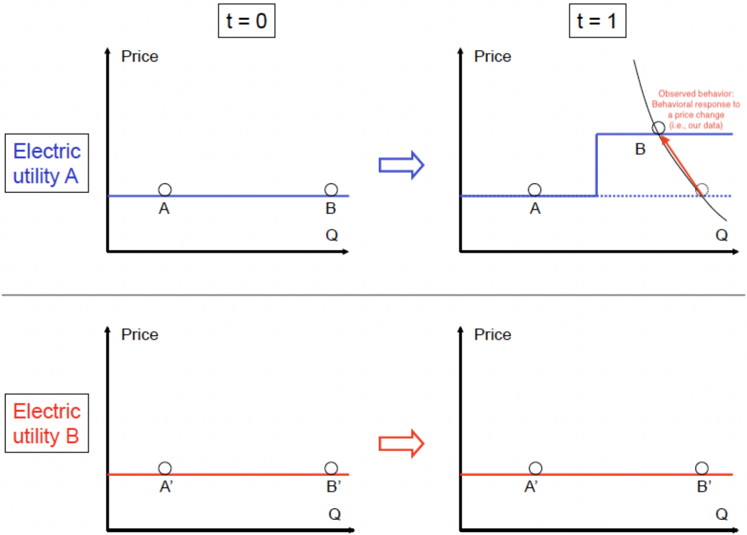

Ito (2014)

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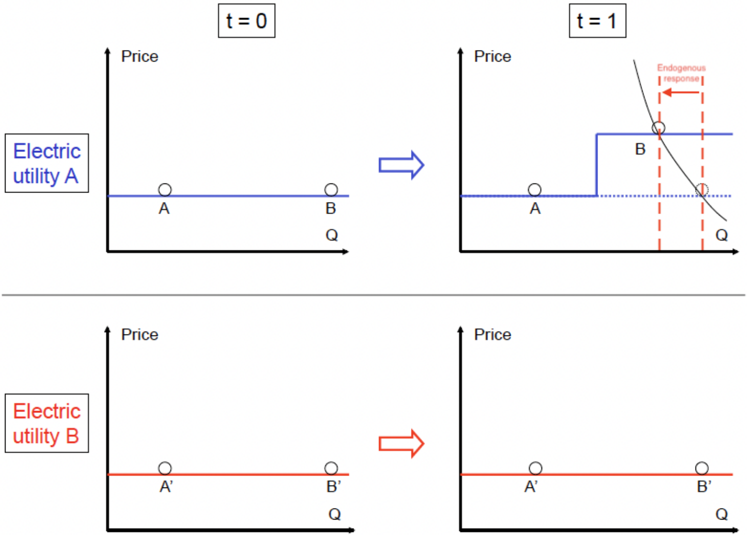

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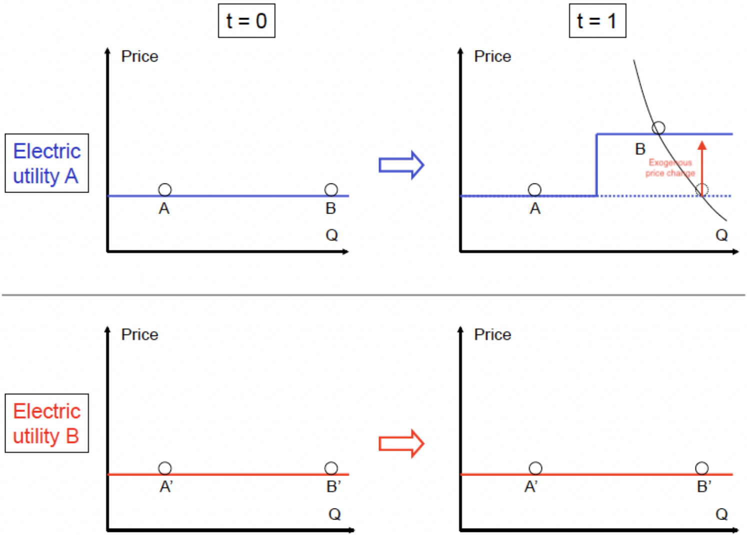


Ito (2014)

- Illustrate situation with point B falling to tier 1
- Identification problem: Our data (observed behavior) is the result of behavioral responses to an exogenous price change
  - But the price we observe after the behavioral response is a function of the behavioral response, too
- Solution: Isolate the exogenous variation in the price change
  - Use x_{it} and plug it into both 1) price schedule at $t = 0$ to get p_0, 2) price schedule at $t = 1$ to get p_1:
  - $\Delta lnp_t^{PI}(x_{it}) = lnp_{t_1}(x_{it}) - lnp_{t_0}(x_{it})$
- x_{it} is the input in both terms, so behavioral response is not driving variation in $\Delta lnp_t^{PI}(x_{it})$
  - Variation in $\Delta lnp_t^{PI}(x_{it})$ is only due to changes in price schedule ($p_{t_1}(\cdot)$ and $p_{t_0}(\cdot)$)

Sufficient Statistics

Sufficient Statistics

- Harberger formula is an approximation
- Hausman (1981) and Hausman and Newey (1995) estimate structural models of demand to estimate exact consumer surplus
- Underscores broader difference between structural and quasi-experimental methodologies
- Modern literature focuses on deriving "sufficient statistic" formulas that can be implemented using quasi-experimental techniques
- Now develop general distinction between structural and sufficient statistic approaches to welfare analysis in a simple model of taxation
  - No income effects (quasilinear utility)
  - Constant returns to production (fixed producer prices)
  - But permit multiple goods (GE)

Sufficient Statistics

- N goods: $x = (x_1, ..., x_N)$; prices $(p_1, ...p_N)$; wealth Z
- Normalize $p_N = 1$ (x_N is numeraire)
- Government levies a tax t on good 1
- Individual takes t as given and solves

$$\max_x u(x_1, ..., x_{N-1}) + x_N$$

$$\text{s.t. } (p_1 + t)x_1 + \sum_{i=2}^{N} p_i x_i = Z$$

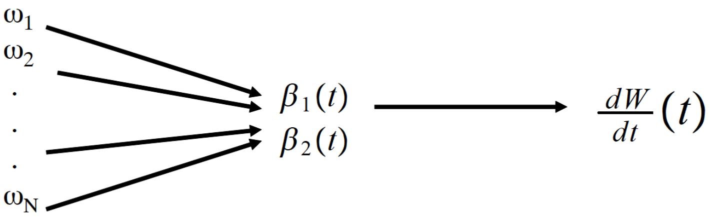
- To measure DWL of tax, define social welfare as sum of individual's utility and tax revenue:

$$W(t) = \{\max_x u(x_1, ..., x_{N-1}) + Z - (p_1 + t)x_1 - \sum_{i=2}^{N-1} p_i x_i\} + tx_1$$

- Goal: measure $\frac{dW}{dt}$ = loss in social surplus caused by tax change

Sufficient Statistics

Primitives	Sufficient Stats.	Welfare Change



ω=preferences, constraints	$\beta = f(\omega,t)$ $y = \beta_1X_1 + \beta_2X_2 + \varepsilon$	dW/dt used for policy analysis
ω not uniquely identified	β identified using program evaluation	

Source: Chetty (2009)

Sufficient Statistics

- Structural method: estimate N good demand system, recover u
  - Ex: Use Stone-Geary to recover preference parameters; then calculate "exact consumer surplus" as in Hausman (1981)
- Alternative: Harberger's deadweight loss triangle formula
  - Private sector choices made to maximize term in red (private surplus)

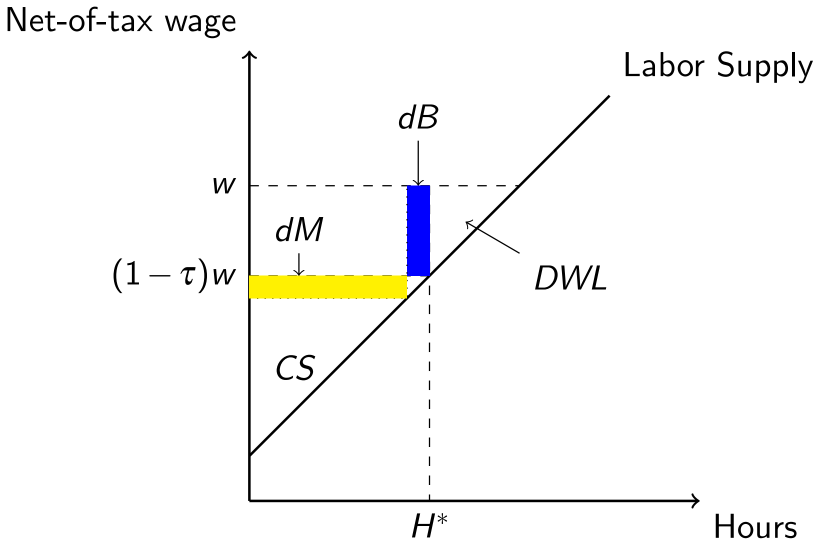
$$W(t) = \left\{ \max_x u(x_1, \ldots, x_{N-1}) + Z - (p_1 + t)x_1 - \sum_{i=2}^{N-1} p_i x_i \right\} + t x_1$$

  - Envelope conditions for $(x_1, \ldots, x_N)$ allow us to ignore behavioral responses ($\frac{dx_i}{dt}$) in term in red, yielding

$$\frac{dW}{dt} = -x_1 + x_1 + t\frac{dx_1}{dt} = t\frac{dx_1}{dt}$$

$\rightarrow \frac{dx_1}{dt}$ is a "sufficient statistic" for calculating $\frac{dW}{dt}$

Sufficient Statistics



Feldstein (1999)

- Following Harberger, large literature in labor estimated effect of taxes on hours worked to assess efficiency costs of taxation
- Feldstein observed that labor supply involves multiple dimensions, not just choice of hours: training, effort, occupation
- Taxes also induce inefficient avoidance/evasion behavior
- Structural approach: account for each of the potential responses to taxation separately and then aggregate
- Feldstein's alternative: elasticity of taxable income with respect to taxes is a sufficient statistic for calculating deadweight loss

Feldstein (1999)

- Government levies linear tax t on reported taxable income
- Agent makes N labor supply choices: $l_1, ...l_N$
- Each choice l_i has disutility $\psi_i(l_i)$ and wage w_i
- Agents can shelter \$$e$ of income from taxation by paying cost $g(e)$
- Taxable Income (TI) is

$$TI = \sum_{i=1}^{N} w_i l_i - e$$

- Consumption is given by taxed income plus untaxed income:

$$c = (1-t)TI + e$$

Feldstein (1999)

- Agent's utility is quasi-linear in consumption:

$$u(c,e,l) = c - g(e) - \sum_{i=1}^{N} \psi_i(l_i)$$

- Social welfare:

$$W(t) = \{(1-t)TI + e - g(e) - \sum_{i=1}^{N} \psi_i(l_i)\} + tTI$$

- Differentiating and applying envelope conditions for l_i $((1-t)w_i = \psi_i'(l_i))$ and e $(g'(e) = t)$ implies

$$\frac{dW}{dt} = -TI + TI + t\frac{dTI}{dt} = t\frac{dTI}{dt}$$

- Intuition: marginal social cost of reducing earnings through each margin is equated at optimum $\rightarrow$ irrelevant what causes change in TI

Feldstein (1999)

- Simplicity of identification in Feldstein's formula has led to a large literature estimating elasticity of taxable income
- But since primitives are not estimated, assumptions of model used to derive formula are never tested
- Chetty (2009) questions validity of assumption that $g'(e) = t$
  - Costs of some avoidance/evasion behaviors are transfers to other agents in the economy, not real resource costs
  - Ex: cost of evasion is potential fine imposed by government

Chetty (2009)

- Individual chooses e (evasion/shifting) and l (labor supply) to

$$\begin{aligned} \max_{e,l} u(c,l,e) &= c - \psi(l) \\ \text{s.t. } c &= y + (1-t)(wl - e) + e - z(e) \end{aligned}$$

- Social welfare is now:

$$\begin{aligned} W(t) &= \{y + (1-t)(wl - e) + e \\ &\quad - z(e) - \psi(l)\} \\ &\quad + z(e) + t(wl - e) \end{aligned}$$

- Difference: $z(e)$ now appears twice in SWF, with opposite signs

Chetty (2009)

- Let $LI = wl$ be the total (pretax) earned income and $TI = wl - e$ denote taxable income
- Exploit the envelope condition for term in curly brackets:

$$\begin{aligned}
\frac{dW}{dt} &= -(wl - e) + (wl - e) + \frac{dz}{de}\frac{de}{dt} + t\frac{d[wl - e]}{dt} \\
&= t\frac{dTI}{dt} + \frac{dz}{de}\frac{de}{dt} \\
&= t\frac{dLI}{dt} - t\frac{de}{dt} + \frac{dz}{de}\frac{de}{dt}
\end{aligned}$$

- First-order condition for individual's choice of e:

$$\begin{aligned}
t &= \frac{dz}{de} \\
\Rightarrow & \frac{dW}{dt} = t\frac{dLI}{dt} \qquad (1)
\end{aligned}$$

- Intuition: MPB of raising e by \$1 (saving \$$t$) equals MPC

Chetty (2009)

- With both transfer cost $z(e)$ and resource cost $g(e)$ of evasion:

$$\begin{aligned}\frac{dW}{dt} &= t\frac{dLI}{dt} - g'(e)\frac{de}{dt} \\ &= t\{\mu\frac{dTI}{dt} + (1-\mu)\frac{dLI}{dt}\} \\ &= -\frac{t}{1-t}\{\mu TI\varepsilon_{TI} + (1-\mu)LI\varepsilon_{LI}\}\end{aligned}$$

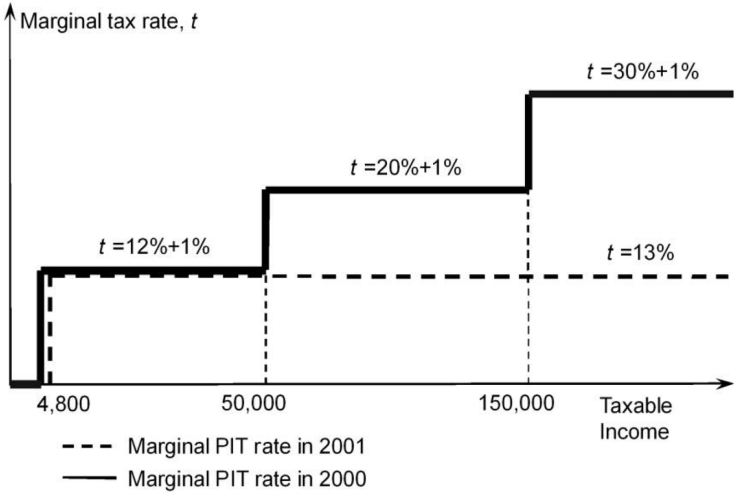
- *DWL* depends on weighted average of taxable income (ε_{TI}) and total earned income elasticities (ε_{LI})
    - Practical importance: even though reported taxable income is highly sensitive to tax rates for rich, efficiency cost may not be large!
- Most difficult parameter to identify: weight μ, which depends on marginal resource cost of sheltering, $g'(e)$

Gorodnichenko, Martinez-Vazquez, and Peter (2009)

- Estimate ε_{LI} and ε_{TI} to implement formula that permits transfer costs
- Insight: consumption data can be used to infer ε_{LI}
- Estimate effect of 2001 flat tax reform in Russia on gap between taxable income and consumption, which they interpret as evasion

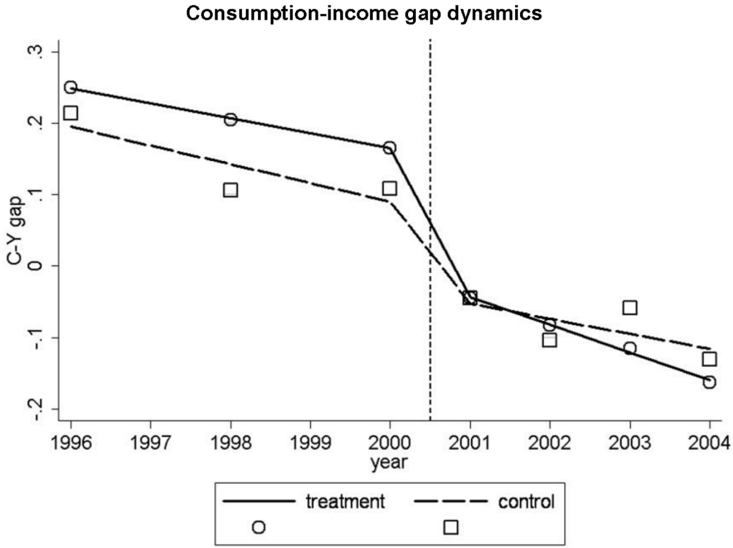
Gorodnichenko, Martinez-Vazquez, and Peter (2009)

Marginal personal income tax rate before and after the reform



Source: Gorodnichenko, Martinez-Vazquez, and Peter 2009

Gorodnichenko, Martinez-Vazquez, and Peter (2009)



Source: Gorodnichenko, Martinez-Vazquez, and Peter 2009

Gorodnichenko, Martinez-Vazquez, and Peter (2009)

- Taxable income elasticity $\frac{dTI}{dt}$ is large, whereas labor income elasticity $\frac{dLI}{dt}$ is not

  $\rightarrow$ Feldstein's formula overestimates the efficiency costs of taxation relative to more general measure for "plausible" $g'(e)$

- Question: could $g'(e)$ be estimated from consumption data itself?

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