

Discussion Section 1 Questions

1. (Budget Constraints) Rachel has a monthly income of \$200, which she spends on cellular telephone minutes (C) and a numeraire good (N) that is priced at \$1. MES Telephone Service offers her two plans: Plan A and Plan B. Under Plan A she pays \$50 per month and can use the telephone for up to 300 minutes with no additional charge. If she makes more calls, then she must pay \$0.10 for each additional minute. Under Plan B she pays \$80 per month and can use the telephone for up to 1200 minutes with no additional charge. If she makes more calls then she must pay \$0.10 for each additional minute. Draw a graph illustrating Rachel's budget constraint.
2. (Condorcet Cycle) Suppose there are three individuals, Rachel, Sam, and Tom, with the following preferences over three goods, A , B , and C :

$$\text{Rachel: } A \succ B \succ C$$

$$\text{Sam: } C \succ A \succ B$$

$$\text{Tom: } B \succ C \succ A$$

Note that the preferences overall indicate that $C \succ A$ (because Sam and Tom both prefer C over A), $A \succ B$ (because Rachel and Sam both prefer A over B), and $B \succ C$ (because Rachel and Tom both prefer B over C). Thus, each individual's preferences are transitive, but the overall preferences of the group are not transitive. Now suppose that Rachel is set with the task of putting together a two-stage vote. She will choose two goods over which the group will vote for the favorite in the first round, and the winner of the first round will then face the remaining good in a vote for the favorite in the second round.

For example, Rachel could choose to hold a vote between A and B in the first round and then the winner of that vote will face a vote between itself and C . The question is: since Rachel likes good A the best, how should she arrange the vote to guarantee that good A is the overall winner after two rounds of voting? (Note that voters are *not* required to vote based on their preferences; they can vote in any way. The preferences refer to the final good that remains after voting ends).

3. (Utilities) Suppose that the utility functions $u(x, y)$ and $v(x, y)$ are related by $v(x, y) = f(u(x, y))$. In other words, the utility function $v(x, y)$ is a function of the utility function $u(x, y)$. In each case, determine whether the function f is a monotonically increasing transformation.

a. $f(u) = \sqrt{3}u$

b. $f(u) = 5,000 - 23u$

c. $f(u) = u - 100,000$

d. $f(u) = \ln(u)$

e. $f(u) = -e^{-u}$

f. $f(u) = \frac{1}{u}$

g. $f(u) = -\frac{1}{u}$

4. (Leontief Utility) Karl's preferences over hamburgers (H) and beer (B) are described by the utility function $U(H, B) = \min\{2H, 3B\}$. His monthly income is I dollars, and he only buys these two goods out of his income. Denote the price of hamburgers by p_H and of beer by p_B .

a. Derive Karl's demand curve for beer as a function of the exogenous variables.

- b. Which affects Karl's consumption of beer more: a one dollar increase in p_H or a one dollar increase in p_B ?

5. (CES Utility) The Constant Elasticity of Substitution (CES) utility function has the form (with only two goods):

$$U(x, y) = [ax^\rho + by^\rho]^{\frac{1}{\rho}}$$

Where a , b , and ρ are exogenous parameters and x and y are the two goods available for purchase. When $\rho = 1$ the utility function is a linear utility function. If $a + b = 1$ then as ρ approaches 0 the utility function converges to the Cobb-Douglas utility function. As ρ approaches $-\infty$ the utility function converges to the Leontief utility function.

- Find the demand functions for x and y in terms of income (M), the parameters (a , b , and ρ), and prices (p_x and p_y). Assume there is an interior solution.
- Prove that if $a + b = 1$ then as ρ approaches 0 the CES utility function converges to the Cobb-Douglas utility function.
- Prove that as ρ approaches $-\infty$ the CES utility function converges to the Leontief utility function.

6. (Borrowing And Lending) Suppose that Elliott makes consumption decisions over two periods. In other words, he chooses how much money to spend in the first period and how much to spend in the second period. Elliott can borrow at a rate r_B and lend at a rate r_L , where $r_L < r_B$. Elliott purchases C_1 units of a composite good (a good that represents all possible goods available for purchase) in period 1 and C_2 units of a composite good in period 2.

- a. Let A denote the basket of goods at which a consumer spends exactly his income each period. Prove that a consumer with a diminishing MRS_{C_1, C_2} will choose to borrow in the first period if at basket A it is such that $MRS_{C_1, C_2} > 1 + r_B$, will choose to lend if at basket A it is such that $MRS_{C_1, C_2} < 1 + r_L$, and will choose to neither borrow nor lend if at basket A it is such that $1 + r_B < MRS_{C_1, C_2} < 1 + r_L$. A graphical proof is acceptable.
- b. Suppose that Elliott earns \$2,000 this month and will earn \$2,200 next month and has a utility function of $U(C_1, C_2) = C_1 C_2$. Suppose that r_L is 5% and r_B is 12%. Draw Elliott's budget set over the two time periods. Will Elliott borrow, lend, or do neither this month?
- c. Suppose that r_B falls to 8%. Draw (on the same graph as part (b)) Elliott's new budget set and determine whether Elliott will borrow, lend, or do neither this month.

7. (Income And Substitution Effects) Melanie purchases two goods, food (f) and clothing (c). She has the utility function $U(f, c) = fc$. Now suppose that she has an income of \$72 per week and that the initial prices of clothing and food is $p_c = 1$ and $p_f = 9$. Melanie wakes up this morning and finds that p_f fell to 4. Find the numerical values of the income and substitution effects on food consumption and graph the results.

8. (Equivalent Variation) Elliott spends his money on books (x) and beer (y). He has the following utility function: $u(x, y) = x^{\frac{2}{3}}y^{\frac{1}{3}}$. The prices of books is $p_x = 2$, the price of beer is $p_y = 2$, and his income is $m = 4$.

Half of his income he earned by working part time at Bubba's Bargain books and the other half he earned by working part time at Bubba's Gym. Further suppose Bubba's Bargain Books closes, decreasing Elliott's income to $m = 2$ and increasing the price of books to $p_x = 4$. What is the equivalent variation?

9. (Elasticity) Some texts define a "luxury good" as a good for which the income elasticity of demand is greater than 1. Suppose that a consumer purchases only two goods. Can both goods be luxury goods? Explain and show your claim algebraically.

10. (Uncertainty) Suppose that I represents income. Your utility function is given by $U(I) = 10I$ as long as I is less than or equal to 300. If I is greater than 300, your utility is given by $U(I) = 100\sqrt{3I}$. Suppose you have a choice between having an income of 300 with certainty and a lottery that makes your income equal to 400 with probability 0.5 and equal to 200 with probability 0.5.

- a. Sketch this utility function
- b. What is the expected payoff of each lottery?
- c. Which lottery do you prefer?
- d. Are you risk averse, risk neutral, or risk loving?

11. (Uncertainty) Elliott has a utility function given by $U(I) = 1 - 10I^{-2}$, where I denotes his monetary payoff from an investment in thousands. Elliott is considering an investment that will give him a payoff of \$10,000 (thus, $I = 10$) with probability 0.7, a payoff of \$8,500 ($I = 8.5$)

with probability 0.2, and a payoff of \$2,000 ($I = 2$) with probability 0.1. It will cost him \$8,000 to make the investment. Should he make the investment?

12. (Market Demand) Suppose the market for rental cars has two segments, business travelers and vacation travelers. The demand curve for rental cars by business travelers is $Q_B = 35 - 0.25P$, where Q_B is the quantity demanded by business travelers (in thousands of cars) when the rental price is P dollars per day. No business customers will rent cars if the price exceeds \$140 per day. The demand curve for rental cars by vacation travelers is $Q_V = 120 - 1.5P$, where Q_V is the quantity demanded by vacation travelers (in thousands of cars) when the rental price is P dollars per day. No vacation customers will rent cars if the price exceeds \$80 per day. There is a monopoly in the rental car industry.

- a. Graph the demand and marginal revenue curves for each segment and draw the market demand and market marginal revenue curves for rental cars.
- b. Describe the market demand curve and the market marginal revenue curve algebraically.
- c. Suppose the monopoly has a constant marginal cost of \$20 per day. Find the profit maximizing prices and quantities if the monopoly can charge different kinds of customers different prices.

13. (Profit Maximization) Consider the following production function: $f(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, where $0 < \alpha < 1$.

- a. Formulate the profit-maximizing problem when the price per unit of output is p and the price of x_1 is w_1 and the price of x_2 is w_2 .
- b. Derive the profit-maximizing conditions.

- c. Now suppose that the pair (x_1, x_2) solves the profit-maximizing conditions and is an interior solution. Multiply the condition for x_1 (from (b)) by x_1 and multiply the condition for x_2 (from (b)) by x_2 . Then add the two conditions together. What does the result tell you about the profit at the profit-maximizing pair (x_1, x_2) ?

14. (Taxes) Suppose that the market is perfectly competitive and that demand is given by

$$Q_d(p_d) = \frac{1}{p_d} \text{ and supply is given by } Q_s(p_s) = p_s^2.$$

- a. What is the equilibrium price and quantity?
- b. Suppose the government imposes a small per-unit tax of size τ . Let p_s be the net price received by suppliers in the market and let p_d be the price paid by consumers (including the tax) in the market. Find the derivatives of p_s and p_d with respect to τ and explain in words what is meant by these expressions.
- c. Using the expressions from (b) find the approximate new prices (p_d and p_s) and the approximate new equilibrium quantity if the government imposes a tax of \$0.30 per unit sold.
- d. Draw a graph of the situation. Label Consumer Surplus, Producer Surplus, Tax Revenue, and Deadweight Loss.
- e. Calculate the approximate change in Consumer Surplus, the approximate change in Producer Surplus, and calculate the approximate Deadweight Loss using your results from (b).

15. (Cost Minimization) Suppose a firm produces output using two inputs, x and y , and has the following isoquant for producing a units of output:

$$f(x, y) = a \quad \text{if } a < x + y \leq (a + 1)$$

For simplicity, assume that $p_x = p_y = 1$. Also assume that a must be an integer. Suppose the firm wants to produce exactly a units of the good and seeks to minimize its cost to produce those units. What problem will the firm run into? Explain.

16. (Cost Function) Suppose a firm has the production function $f(x, y) = \min\{3x, 4y\}$ and wants to produce exactly z units of output. What is the cost function associated with this production function? In other words, find the function $c(p_x, p_y, z)$ that is a function of p_x , p_y , and z that will tell you how much it will cost to produce z units at prices p_x and p_y .

17. (Cost Function) Consider the production function $f(L, K) = 2L^{\frac{1}{2}}K^{\frac{1}{2}}$. The price of L is w , the price of K is r , and the price of the output good is 1. The market is perfectly competitive.

- a. Suppose that in the short run capital is fixed at $K = K_0 > 0$. Find the short run cost function.
- b. Find the long run demand functions for L and K as functions of Q , w , and r .
- c. Find the long run cost function, C_{LR} , as a function of Q , w , and r .
- d. Find the relationship between w and r in the long run in this market. In other words how are w and r locked together, in some sense, to make sure that the market reaches the long run equilibrium? This is not to say that the firms can choose w and r , this just means that w and r must be related in some way to reach the long run equilibrium. (Hint: there is a property of the market that you should use here.)

18. (Input Demands) Consider the production function $f(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\}$. Find the demand functions for the inputs x_1 and x_2 in terms of the prices of the inputs, w_1 and w_2 , and the level of output, y .

19. (Cost Curves) Consider the production function $f(L, K) = (\sqrt{L} + \sqrt{K})^2$. The price of L is 2 and the price of K is 1.

- Find the long run demand functions for L and K as a function of output, Q .
- Find the long run cost function, C_{LR} , as a function of Q .
- Find the short run cost function, C_{SR} , when capital is fixed at $K = 9$.
- Graph the short run cost function when $K = 9$ and the long run cost function

20. (Market Supply) Suppose a firm has the production function $f_i(K, L) = L_i^{\frac{3}{7}} K_i^{\frac{4}{7}}$. In the short-run K_i is fixed at $K_i = \bar{K} = 2$ for all firms. There are 100 identical firms. The price of L_i is w , the price of K_i is r , and the price of output is p for all firms.

- Find each individual firm's short-run supply curve as a function of p , w , and r .
- Suppose that market labor supply is $LS(w) = 50w$. Find the total market short-run supply curve as a function of p only.
- Find the market elasticity of supply

21. (Monopoly) Suppose a monopolist supplies rental cars. There are two types of consumers who rent cars: vacationers and businessmen. Businessmen have a demand curve of $Q_B = 1 - \frac{1}{3}P$ and vacationers have a demand curve of $Q_V = 4 - 2P$. Assume there is a single consumer of

each type in the economy. The monopolist cannot observe the individual types of consumers and must set a single price for the entire market.

- a. Find the mathematical expression for the market demand curve.
- b. Find the mathematical expression for the market marginal revenue curve
- c. If the monopolist has a constant marginal cost of 1.8 what is the price the monopolist will charge and what is the monopolist's profit?
- d. If the monopolist has a constant marginal cost of 1.5 what is the price the monopolist will charge and what is the monopolist's profit?
- e. Find the level of marginal cost, m^* , that will make the monopolist indifferent between serving both businessmen and vacationers and serving only the businessmen.

21. (Stackelberg Competition) Find the Stackelberg duopoly equilibrium quantities and equilibrium price when both firms have the inverse market demand function $p(Q) = (1 + Q)^{-3}$. Assume that firm 1 is the leader and firm 2 is the follower. Each firm has zero cost.

22. (Cournot Competition) Find the Cournot duopoly equilibrium quantities when both firms face the market demand function $Q = 4 - p$, firm 1 has a cost of $c_1 = 1$, and firm 2 has a cost of $c_2 = 3$. Additionally, what is the equilibrium price and how much profit does each firm make in equilibrium?

23. (Bertrand Competition) What are the profits of each firm in Bertrand competition when market demand is $Q = 5 - p$, firm 1 has a constant marginal cost $c_1 = 1$, and firms 2 and 3 have constant marginal costs $c_2 = c_3 = 2$.

24. (Two-Part Tariff) Suppose that a fraction, f , of consumers have demand $Q = 3 - p$ and the remaining fraction, $1 - f$, of consumers have demand $Q = 2 - p$. The market is a monopoly. Suppose that the monopoly cannot observe the types of specific individuals, but does know the fractions, f and $1 - f$, of consumers with each type of demand function. The monopoly can only set one two-part tariff and has a constant marginal cost of 1. Find the optimal two-part tariff, the consumer surplus for both types of consumers, the quantity consumed by both types of consumers, and the profit of the firm. (Be careful!)

25. (Edgeworth Box) Consider an economy with two individuals, Josh and Ema. There are two goods, x and y , in fixed quantities \underline{x} and $\underline{y} = k\underline{x}$.

- a. Suppose Josh and Ema have the same Cobb-Douglas preferences described by the utility function $u(x, y) = \ln(x) + \ln(y)$. Find the expression for MRS_J and MRS_E . How will the ratio of $\frac{y}{x}$ for Josh ($\frac{y_J}{x_J}$) along the contract curve compare to Emma's ratio ($\frac{y_E}{x_E}$)?
- b. Prove that the contract curve is the diagonal of the Edgeworth box.

26. (Quasilinear Utility) True, false, or uncertain: In an Edgeworth box economy where both consumers have continuous, strictly convex, and quasilinear preferences (where good 1 is the numeraire good) then any two Pareto-efficient allocations in the interior of the box must involve the same consumptions of the second good.

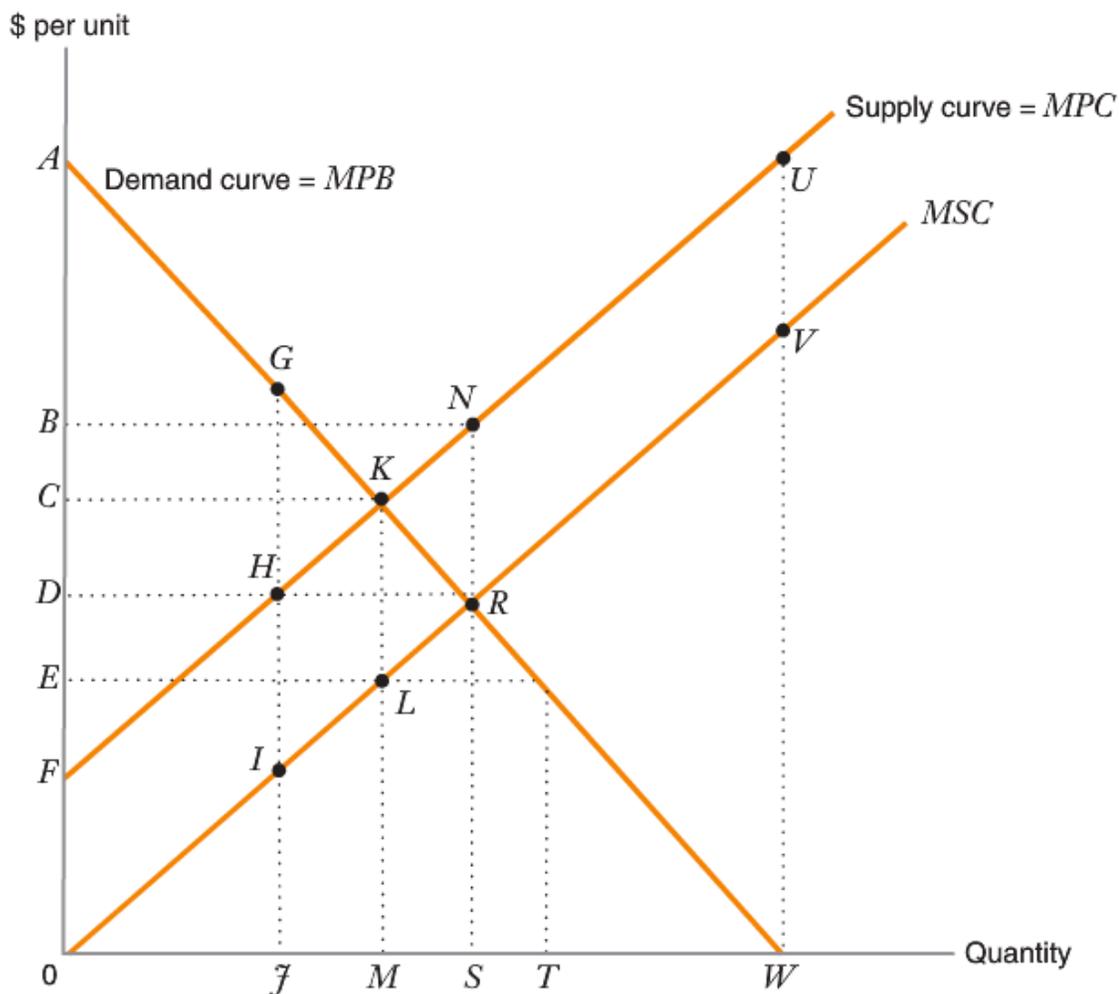
27. (Exchange) Suppose Adam Smith and Thomas Jefferson meet at teatime. Smith brought with him 1 kilo of tea (T) and Jefferson brought with him 1 kilo of Virginia tobacco (V). Their utilities functions are:

$$U_{TJ} = T + 2V$$

$$U_{AS} = 2T + V$$

- a. Draw an Edgeworth box of this situation. Put Thomas Jefferson in the bottom left corner and Adam Smith in the upper right corner. Put V on the horizontal axis and T on the vertical axis. Indicate the contract curve and the competitive equilibrium allocation.
- b. Repeat (a) with Smith bringing $\frac{1}{2}$ of a kilo of tea and Jefferson bringing 1 kilo of Virginia tobacco and 1 kilo of tea.
- c. What is surprising about the results of (b)?

28. (Externalities) The graph below shows conditions in a perfectly competitive market in which there is some sort of externality. In this market a consumer purchases at most one unit of the good. There are many such consumers and they have different maximum willingnesses to pay. Assume the graph is drawn to scale and that due to high transaction costs it is impossible for those affected by the externality to resolve it privately without government intervention.



- a. What type of externality is present in this market: positive or negative?
- b. What is the maximum level of social surplus that is potentially attainable in this market?
- c. What is the deadweight loss that arises in a competitive equilibrium in this market?
- d. What is the magnitude of a per-unit subsidy that would enable this market to attain the socially efficient outcome?
- e. Indicate whether the following would increase, decrease, or not affect social efficiency relative to the competitive equilibrium:
 - i. A per-unit subsidy equal to F given to consumers who purchase the good.

- ii. The government imposes a price ceiling that sets the maximum price of the good equal to D .
- iii. The government imposes a tax equal to NR on consumers who *do not* purchase the good.

28. (Public Goods) There are two citizens in Sim City. The first citizen is willing to pay \$30 per hour for each hour up to $Q = 8$ and not willing to pay for any hours above 8. The second citizen has the inverse demand curve $P = 60 - 3Q$.

- a. Suppose the marginal cost of public broadcasting is $MC = 15$. What is the economically efficient level of public broadcasting?
- b. Repeat part (a) with $MC = 45$

29. (Externalities) Suppose that two agents are deciding how fast to drive their cars. Agent i chooses speed x_i and gets utility $u_i(x_i)$ from this choice; we assume that $u_i'(x_i) > 0$. However, the faster the agents drive, the more likely it is that they are involved in a mutual accident. Let $p(x_1, x_2)$ be the probability of an accident, assumed to be increasing in each argument, and let $c_i > 0$ be the cost that the accident imposes on agent i . Assume that each agent's utility is linear in money, that the $u_i(x_i)$ and $p(x_1, x_2)$ functions are such that the first order conditions are necessary and sufficient for solutions to the utility maximization and social planner's problems, and that due to high transaction costs it is impossible for those affected by the externality to resolve it privately without government intervention.

- a. Show that each agent has an incentive to drive too fast from the social point of view.

- b. If agent i is fined an amount t_i in the case of an accident how large should t_i be to internalize the externality?
- c. If the optimal fines are being used what are the total costs, including fines, paid by the agents? How does this compare to the total cost of the accident?
- d. Suppose now that agent i gets utility $u_i(x_i)$ only if there is no accident. What is the appropriate fine in this case?